# Factoring Polynomial Expressions Lesson #5:
## Difference of Squares

***Investigation***

**a)** Complete the following using the trinomial factoring method from the previous lessons.

| | Sum | Product | Integers | Polynomial | Factored Form |
|---|---|---|---|---|---|
| **i)** | –6 | –16 | -8, 2 | $x^2 - 6x - 16$ | $(x-8)(x+2)$ |
| **ii)** | –15 | –16 | -16, 1 | $x^2 - 15x - 16$ | $(x-16)(x+1)$ |
| **iii)** | 0 | –16 | -4, 4 | $x^2 + 0x - 16 = x^2 - 16$ | $(x-4)(x+4)$ |
| **iv)** | 0 | –64 | -8, 8 | $x^2 - 64$ | $(x-8)(x+8)$ |
| **v)** | 0 | –25 | -5, 5 | $x^2 - 25$ | $(x-5)(x+5)$ |

**b)** The third row in a) shows that the factored form of $x^2 - 16$ is $(x - 4)(x + 4)$.
Use the pattern from the last three rows to factor the following.

**i)** $x^2 - 9 =$ $(x-3)(x+3)$

**ii)** $x^2 - 49 =$ $(x-7)(x+7)$

**iii)** $x^2 - 36 =$ $(x-6)(x+6)$

**iv)** $x^2 - 1 =$ $(x-1)(x+1)$

**v)** $a^2 - 100 =$ $(a-10)(a+10)$

**c)** Extend the procedure from above to factor $m^2 - n^2$.
Verify your answer by expanding the factored form.

$m^2 - n^2 = (m-n)(m+n)$
$= m^2 - mn + mn - n^2$
$= m^2 - n^2$

**d)** Consider the expansion $(x - y)(x + y) = x^2 + bx + c$.

**i)** Explain why the value of $b$ is zero.

$xy - xy = 0 = bx$

**ii)** Express $c$ in terms of $y$.

$c = -y^2$

## *Difference of Squares*

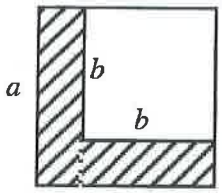
The examples on the previous page are trinomials of the form $x^2 + bx + c$, where $b = 0$ and $c$ is the negative of a square number.

This results in a **difference of squares** such as $x^2 - 25$, $x^2 - 100$, etc.

To factor a difference of squares we can use the identity:

$$a^2 - b^2 = (a - b)(a + b)$$

The identity $a^2 - b^2 = (a - b)(a + b)$ can be illustrated in the following diagram.




Shaded area = $a^2 - b^2$


Shaded area = $(a - b)(a + b)$

*The shaded area on the left is cut along the dotted line and rearranged to form the diagram on the right.*

*The shaded area on the left is represented by $a^2 - b^2$ and the shaded area on the right is represented by $(a - b)(a + b)$.*

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Factor the following polynomials using the difference of squares method.

**a)** $a^2 - 4$
$(a-2)(a+2)$

**b)** $t^2 - 144$
$(t-12)(t+12)$

**c)** $x^2 - y^2$
$(x-y)(x+y)$

**d)** $p^2 - 7^2$
$(p-7)(p+7)$


Note that it is not possible to factor a sum of squares like $x^2 + 4$, i.e. $x^2 + 0x + 4$.
It is not possible to find two integers whose product is positive and whose sum is zero.

In the identity $a^2 - b^2 = (a - b)(a + b)$ we can replace $a$ and/or $b$ by numbers, variables, monomials and even polynomials.

For example, $4x^2 - 25$ can be written as $(2x)^2 - (5)^2$ and can be factored using the above identity with $a = 2x$ and $b = 5$.

$$4x^2 - 25 = (2x - 5)(2x + 5)$$

$9m^2 - 4n^2$ can be written as $(3m)^2 - (2n)^2$, and can be factored using the above identity with $a = 3m$ and $b = 2n$.

$$9m^2 - 4n^2 = (3m - 2n)(3m + 2n)$$

The factoring above can be verified by expanding the product of the factors.

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*Class Ex. #2*

Factor, if possible, using the difference of squares method.

**a)** $16t^2 - 49$
$(4t - 7)(4t + 7)$

**b)** $81a^2 - 1$
$(9a - 1)(9a + 1)$

**c)** $100 - y^2$
$(10 - y)(10 + y)$

**d)** $36p^2 - 25q^2$
$(6p - 5q)(6p + 5q)$

**e)** $4x^2 + 25$
$(2x + 5)(2x - 5)$

**f)** $64 - 9a^2b^2$
$(8 - 3ab)(8 + 3ab)$

*Class Ex. #3*

The floor of an international doubles squash court is rectangular with an area of $25a^2 - b^2$ square feet.

**a)** Write expressions for the length and width of the floor.

$l = 5a - b$ $\quad w = 5a + b$

**b)** The perimeter of the floor is 140 feet. Determine the length and width of the floor if the length is 1.8 times the width.

Part 1:
$P = 2l + 2w$
$140 = 2(5a - b) + 2(5a + b)$
$140 = 10a - 2b + 10a + 2b$
$140 = 20a$
$a = 7$

Part 2:
$5a + b = 1.8(5a - b)$
$5a + b = 9a - 1.8b$
$2.8b = 4a$
$b = 4(7) \div 2.8$
$b = 10$

Part 3:
$l = 5a - b$
$= 5(7) - 10$
$= 25$ ft

$w = 5a + b$
$= 5(7) + 10$
$= 45$ ft

## *Difference of Squares involving a Common Factor*

The first step in factoring any polynomial expression should be to determine if we can remove a common factor.


Factor the following polynomials by first removing the greatest common factor.

**a)** $2a^2 - 50$ $2(a-5)(a+5)$

**b)** $3x^2 - 12y^2$ $3(x^2-4y^2)$ $3(x-2y)(x+2y)$

**c)** $144p^2q^2 - 4$ $4(36p^2q^2-1)$ $4(6pq-1)(6pq+1)$

**d)** $3x^3 - 27x$ $3x(x^2-9)$ $3x(x-3)(x+3)$

**Complete Assignment Questions #1 - #14**

# Assignment

**1.** Complete the following by determining the missing factor.

**a)** $x^2 - 36 = (x - 6)($ $)$ **b)** $c^2 - 121 = (c + 11)($ $)$ **c)** $j^2 - k^2 = (j - k)($ $)$

**2.** Factor the following polynomials using a difference of squares.

**a)** $x^2 - 49$ **b)** $x^2 - 1$ **c)** $x^2 - 15^2$ **d)** $x^2 - 400$

**3.** Explain how factoring a difference of squares in one variable can be regarded as a special case of factoring trinomials by inspection.

**4.** Factor where possible.

**a)** $m^2 - n^2$ $(m-n)(m+n)$

**b)** $c^2 - 7^2$ $(c-7)(c+7)$

**c)** $1 - k^2$ $(1-k)(1+k)$

**d)** $g^2 - 64h^2$ $(g-8h)(g+8h)$

**e)** $25x^2 - 144$ $(5x-12)(5x+12)$

**f)** $16a^2 - 9b^2$ $(4a-3b)(4a+3b)$

**g)** $4x^2 + z^2$ not factorable

**h)** $121a^2 - 36b^2$ $(11a-6b)(11a+6b)$

**i)** $49 - 4h$ not factorable

**j)** $100 - 81b^2$ $(10-9b)(10+9b)$

**k)** $1 - 25z^2$ $(1-5z)(1+5z)$

**l)** $225a^2 - b^2$ $(15a-b)(15a+b)$

**m)** $169z^2 - 4q^2$ $(13z-2q)(13z+2q)$

**n)** $256 - y^2$ $(16-y)(16+y)$

**o)** $t^2 + 36z^2$ not factorable

**p)** $49a^2 - 400$ $(7a-20)(7a+20)$

**5.** The floor of a classroom is rectangular with an area of $81m^2 - 4n^2$ square metres.

**a)** Write expressions in $m$ and $n$ for the length and width of the floor.

$l = 9m - 4n$ $\quad w = 9m + 4n$

**b)** If the perimeter of the floor is 72 metres, form an equation in $m$ and $n$ and solve for $m$.

$72 = 2(9m - 4n) + 2(9m + 4n)$
$72 = 18m - 8n + 18m + 8n$
$72 = 36m$
$m = 2$

**c)** Determine the length and width of the floor if the length is 25% greater the width.

$9m + 4n = 1.25(9m - 4n)$
$9m + 4n = 11.25m - 5n$
$9n = 2.25m$
$n = \frac{2.25(2)}{9}$
$n = .5$

$l = 9m - 4n$
$= 9(2) - 4(.5)$
$= 16\ m$
$w = 9m + 4n$
$= 9(2) + 4(.5)$
$= 20\ m$

**6.** Factor.

**a)** $8x^2 - 32$

**b)** $4a^2 - 100y^2$

**c)** $3t^2 + 27s^2$

**d)** $7x^2 - 7y^2$

**e)** $9a^2b^2 - 36$

**f)** $8 - 50p^2q^2$

**g)** $xy^2 - x^3$

**h)** $20a^2b^2 - 5a^4b^4$

**7.** Factor.

**a)** $a^2b^2 - 9$
$(ab - 3)(ab + 3)$

**b)** $c^2 - d^2e^2$
$(c - de)(c + de)$

**c)** $100x^2 - y^2z^2$
$(10x - yz)(10x + yz)$

**d)** $p^2q^2 - r^2s^2$
$(pq - rs)(pq + rs)$

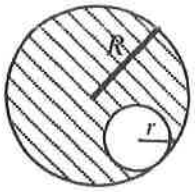
**e)** $25x^2y^2 - 1$
$(5xy - 1)(5xy + 1)$

**f)** $c^2d^2 - 4f^2$
$(cd - 2f)(cd + 2f)$

**g)** $4x^2a^2 - 49z^2t^2$
$(2xa - 7zt)(2xa + 7zt)$

**h)** $16a^2c^2 - 225b^2d^2$
$(4ac - 15bd)(4ac + 15bd)$

**8.** The diagram shows a circle of radius $R$ with a circle of radius $r$ removed.




**a)** Write an expression for the shaded area.

$A = \pi R^2 - \pi r^2$

**b)** Write the expression in a) in factored form.

$A = \pi (R^2 - r^2)$

**c)** Determine the shaded area (as a multiple of $\pi$) if $R = 8.5$ and $r = 1.5$. Do not use a calculator.

$A = \pi(8.5^2 - 1.5^2)$
$= \pi(8.5 - 1.5)(8.5 + 1.5)$
$= \pi(7)(10) = \pi(70)$

✳ **9.** The expression $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$ occurs in physics.

**a)** Write the expression in factored form.

$\frac{1}{2}m(v^2 - u^2)$

**b)** Determine the value of the expression when $m = 10$, $v = 75$, and $u = 25$. Do not use a calculator.

$= \frac{1}{2}10(75^2 - 25^2)$
$= 5(75-25)(75+25)$
$= 5(50)(100)$
$= 25000$

**10.** Consider the following in which each letter represents a whole number.

$64x^2 - y^2 = (Hx - y)(Hx + y)$ $\quad H = 8$

$16x^2 - 4 = C(Ix + 1)(Ix - 1)$ $\quad 4 = C \quad 4x^2 - 1 \quad 2 = I$

$7x^2 - 252y^2 = P(x - Ey)(x + Ey)$ $\quad x^2 - 36y^2 \quad 7(x-6y)(x+6y)$

$Lx^2 - Ny^2 = (3x - 5y)(Sx + Ay)$ $\quad 9x^2 - 25y^2$

Determine the value of each letter and hence name the country represented by the following code.

| (4) | (8) | (2) | (9) | (6) |
|---|---|---|---|---|
| C | H | I | L | E |

**11.** Susan was showing Rose how the difference of squares method can be used to multiply certain numbers without using a calculator. She showed Rose the following:

$$38 \times 42 = (40-2)(40+2) = (40^2 - 2^2) = (1600 - 4) = 1596$$

**a)** Use the above process to evaluate:

i) $27 \times 33$

ii) $61 \times 59$

**b)** Explain why this process cannot be used to determine the product $66 \times 72$.

**c)** Make up your own multiplication question which can be answered using this process.

**Multiple Choice**

**12.** One factor of $16 - 4m^2$ is

$4(4-m^2)$

$4(2-m)(2+m)$

**A.** $4 - m$
**B.** $8 - 2m$
**C.** $4 + m$
**D.** $2 + m$

**13.** Given that $x^2 - y^2 = 45$ and $x + y = 9$, the value of $x$ is

$(x-y)(x+y) = 45$
$9(x-y) = 45$
$x - y = 5$
$x + y = 9$
$2x = 14$
$x = 7$

**A.** 2
**B.** 5
**C.** 7
**D.** impossible to determine

**Numerical Response**

**14.** $3x + 2y$ is a factor of the binomial $a^2x^2 - b^2y^2$.
The value of $a^2 + b^2$ is _____.
(Record your answer in the numerical response box from left to right)

## Answer Key

**1.** **a)** $(x + 6)$ **b)** $(c - 11)$ **c)** $(j + k)$

**2.** **a)** $(x - 7)(x + 7)$ **b)** $(x - 1)(x + 1)$ **c)** $(x - 15)(x + 15)$ **d)** $(x - 20)(x + 20)$

**3.** A difference of squares can be regarded as a trinomial of the form $x^2 + bx + c$ in which $b = 0$ and $c$ is negative. We need to find two numbers which multiply to $c$ and add to zero.

**4.** **a)** $(m - n)(m + n)$ **b)** $(c - 7)(c + 7)$ **c)** $(1 - k)(1 + k)$
**d)** $(g - 8h)(g + 8h)$ **e)** $(5x - 12)(5x + 12)$ **f)** $(4a - 3b)(4a + 3b)$
**g)** not factorable **h)** $(11a - 6b)(11a + 6b)$ **i)** not factorable using whole number exponent.
**j)** $(10 - 9b)(10 + 9b)$ **k)** $(1 + 5z)(1 - 5z)$ **l)** $(15a + b)(15a - b)$
**m)** $(13z - 2q)(13z + 2q)$ **n)** $(16 - y)(16 + y)$ **o)** not factorable **p)** $(7a + 20)(7a - 20)$

**5.** **a)** $(9m + 2n)$ metres, $(9m - 2n)$ metres **b)** $2(9m + 2n) + 2(9m - 2n) = 72$, $m = 2$
**c)** Length = 20 metres, Width = 16 metres.

**6.** **a)** $8(x - 2)(x + 2)$ **b)** $4(a - 5y)(a + 5y)$ **c)** $3(t^2 + 9s^2)$ **d)** $7(x - y)(x + y)$
**e)** $9(ab - 2)(ab + 2)$ **f)** $2(2 - 5pq)(2 + 5pq)$ **g)** $x(y - x)(y + x)$ **h)** $5a^2b^2(2 - ab)(2 + ab)$

**7.** **a)** $(ab - 3)(ab + 3)$ **b)** $(c - de)(c + de)$ **c)** $(10x - yz)(10x + yz)$
**d)** $(pq - rs)(pq + rs)$ **e)** $(5xy - 1)(5xy + 1)$ **f)** $(cd - 2f)(cd + 2f)$
**g)** $(2xa - 7zt)(2xa + 7zt)$ **h)** $(4ac - 15bd)(4ac + 15bd)$

**8.** **a)** $A = \pi R^2 - \pi r^2$ **b)** $\pi(R - r)(R + r)$ **c)** $70\pi$

**9.** **a)** $\frac{1}{2}m(v - u)(v + u)$ **b)** 25 000 **10.** CHILE

**11.** **a)** **i)** 891 **ii)** 3599
**b)** $66 \times 72$ expressed as a difference of squares ($69^2 - 3^2$) cannot easily be evaluated without a calculator or long multiplication.

**12.** D **13.** C **14.**

| | | | |
|---|---|---|---|
| 1 | 3 | | |